The EEG Problem with Uncertain Conductivity

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I. Introduction

Electroencephalography (EEG) is one of the most influential tools in the diagnosis of epilepsy and seizures, as it provides a record of ongoing electrical activity in the brain. The electrodes, connected to the EEG machine, measure signals produced by electrical discharge of neurons in the related areas of the brain. The total electric current $\mathbf{J}$ in the head can be partitioned into two flows: a primary current $\mathbf{J}^p$ related to neural sources, and an ohmic volume current $\mathbf{J}^v$ that results from the effect of the electric field in the volume: $\mathbf{J} = \mathbf{J}^p + \mathbf{J}^v = \mathbf{J}^p + \sigma\mathbf{E} = \mathbf{J}^p - \sigma\nabla V$, where V is the electric potential. A widely used approximation of the neural activity of patients suffering from epilepsy is the representation of the primary current as an electric dipole with dipole moment $\mathbf{d}$ located at $\mathbf{r}_d$ inside the cortex; $\mathbf{J}^p(\mathbf{r}) = \mathbf{d}\delta_{\mathbf{r}_d}(\mathbf{r})$. Since the total current is divergence free and no current flows outside the head H, we obtain

$$\boldsymbol{\nabla} \cdot (\sigma\nabla V) = \boldsymbol{\nabla} \cdot \mathbf{d}\delta_{\mathbf{r}_d}, \quad \text{in} \quad H \tag{1}$$

$$\boldsymbol{\nu} \cdot \sigma\nabla V = 0, \quad \text{on} \quad \partial H \tag{2}$$

where σ is the electric conductivity (EC) and $\boldsymbol{\nu}$ the outward unit normal on ∂H. We will use a *spherical head model* with three layers: brain (radius .87), skull (.92) and scalp (1). A typical assumption is that brain and scalp conductivity are equal, therefore we wil work with the *conductivity ratio* $X = \sigma_{\text{brain}}/\sigma_{\text{skull}}$.

The problem we are faced with is the EC changes from individual to individual and depends on physiological processes. Moreover one can not measure this non invasively, e.g. not preoperatively. Fortunately, one does know upper and lower bounds on the EC values; $X \overset{d}{=} \mathrm{U}[.0051, .0388]$. Also dipole location and moment are treated stochastic. We perform a sensitivity and correlation analysis of EEG sensors.

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II. Methods

Solving the EEG equations at the scalp results in a forward model for the 27 sensor values $\mathbf{S} = \mathrm{L}(\mathbf{r}_d, X) \cdot \mathbf{d}$, where L is the *lead field* matrix. For dealing with the uncertainties we employ *Polynomial Chaos* (PC). It expands a random process $Y(\omega)$, in a similar way as Fourier expansion, in an orthonormal polynomial base $\{\lambda_j(\boldsymbol{\xi}(\omega))\}$ with random arguments $\boldsymbol{\xi}$. The input is written in terms of seven uniform random variables $\boldsymbol{\xi}_{\mathbf{d}}$, $\boldsymbol{\xi}_{\mathbf{r}_d}$ and ξ_X. Propagating this expressions through the lead field model, we obtain the ith sensor response $S_i(\boldsymbol{\xi}) = \sum_{j=0}^{3431} V_{ij}\lambda_j(\boldsymbol{\xi})$ with $V_{ij} = \mathbb{E}[S_i\lambda_j]$ calculated by a sparse grid integration scheme.

III. Results and Conclusions

On average, we observe the least influenced electrodes along the great longitudinal fissure. Also, sensors located closer to a dipole source, are of greater influence to a change in conductivity. The highly influenced sensors were on average located temporal. This was also the case in the correlation analysis. Sensors in the temporal parts of the brain are highly correlated. Whereas the sensors in the occipital and lower frontal region, though they are close together, are not so highly correlated as in the temporal regions.